

## Using Discrete Wavelet Transform and Wiener filter for Image De-noising

By

**Mohammed M. Siddeq**

Software Engineering Depart.  
Technical College / Kirkuk

**Dr. Sadar Pirkhider Yaba**

Physics Depart.  
Science Education College  
Salahhaden University

### Abstract

In this paper we proposed an algorithm for image de-noising based on; the two level discrete wavelet transform (DWT), and Wiener filter, also this paper describe estimate noise power. At first The DWT transform noisy image into sub-bands, consist of low-frequency and high-frequencies, and then estimate noise power for each sub-band. The noise power is computed through two important computations; compute square of variance for each sub-band then compute the mean of the variance. After compute the variance apply the wiener filter on each sub-band by using local window  $n \times n$ , finally perform inverse DWT to obtain de-noised image. Our algorithm tested on the two color images and also compared with Normal Shrink filter and Wiener filter.

الخلاصة:

في هذا البحث نقترح خوارزمية للأزالة التشويه من الصورة مستندة على "DWT" و "Wiener filter"، و أيضا نوضح في هذا البحث تخمين قوة التشويه في الصورة المشوهة. في البداية نطبق "DWT" على الصورة المشوهة لتوليد أربعة مصفوفات منفصلة؛ المصفوفة الأولى تحتوي على قيم الصورة المشوهة ولكن بشكل مصغر. أما المصفوفات الثلاثة المتبقية تحتوي على تفاصيل الصورة وتحتوي على قوة التشويه. في المرحلة الثانية من هذه الخوارزمية نقوم بحساب قوة التشويه في الصورة باستخدام معادلتين موضحة في هذا البحث. والمرحلة الثالثة نطبق خوارزمية "Wiener filter" على كل مصفوفة منفصلة باستخدام نافذة صغيرة. وأخيرا نطبق العملية العكسية لـ "DWT" للحصول على صورة محسنة. في هذه الخوارزمية نوضح عملية المقارنة مع طريقتين مستخدمة في أزالة التشويه

**Keywords** \ Discrete Wavelet Transform, Wiener filter, Estimates noise power

### 1. Introduction

An image is often corrupted by noise in its acquisition and transmission. Image de-noising is used to remove the additive noise while retaining as much as possible the important signal features. In the recent years there has been a fair amount of research on wavelet thresholding and threshold selection for signal de-noising [1,2-4], because wavelet provides an appropriate basis for separating noisy signal from the image signal. The motivation is that as the wavelet transform is good at energy compaction, the small coefficient is more likely due to noise and large coefficient due to important signal features [5]. These small coefficients can be thresholded without affecting the significant features of the image. Thresholding is a simple non-linear technique, which operates on one wavelet coefficient at a time. In its most basic form, each coefficient is thresholded by comparing against threshold, if the coefficient is smaller

than threshold, set to zero; otherwise it is kept or modified. Replacing the small noisy coefficients by zero and inverse wavelet transform on the result may lead to reconstruction with the essential signal characteristics and with less noise. Since the work of Donoho & Johnstone [6,7,8], there has been much research on finding thresholds, however few are specifically designed for images. In this paper, a near optimal threshold estimation technique for image de-noising is proposed which is subband dependent i.e. the parameters for computing the threshold are estimated from the observed data, one set for each subband.

Spatial filters have long been used as the traditional means of removing noise from images and signals. These filters usually smooth the data to reduce the noise, but, in the process, also blur the data [8,9]. In the last decade, several new techniques have been developed that improve on spatial filters by removing the noise more effectively while preserving the edges in the data. Some of these techniques borrow ideas from partial differential equations and computational fluid dynamics. Other techniques combine impulse removal filters with local adaptive filtering in the transform domain to remove not only white & mixed noise, but also their mixtures [10,12]. A different class of methods exploits the decomposition of the data. In this paper we introduce an algorithm for image de-noising by using two level discrete wavelet transform and adaptive filter (i.e. Wiener filter), the idea for our algorithm decompose a noisy image into low frequency and high-frequencies sub-bands. Each sub-band filtered by using adaptive filter, finally perform inverse DWT transform to obtain a de-noised image.

## 2. Discrete Wavelet Transform (DWT)

The Discrete Wavelet Transform (DWT) of image signals produces a non-redundant image representation, which provides better spatial and spectral localization of image formation, compared with other multi-scale representations such as "Gaussian" and "Laplacian" pyramid. Recently, discrete wavelet transform has attracted more and more interest in image de-noising [2-4]. The DWT can be interpreted as signal decomposition in a set of independent, spatially oriented frequency channels. A signal is passed through two complementary filters and emerges as two signals, approximation and Details. This is called *decomposition*. The components can be assembled back into the original signal without loss of information. This process is called *reconstruction* [7]. An image can be decomposed into a sequence of different spatial resolution images using DWT. In case of a 2D image, a 2 level decomposition can be performed resulting different frequency bands namely, LL, LH, HL and HH as shown in Figure-1. These are also known by other names, the sub-bands may be respectively called first average image, horizontal fluctuation, vertical fluctuation and the diagonal fluctuation. The sub-image LL is formed by computing the trends along rows of the image followed by computing trends along its columns. In the same manner, fluctuations are also created by computing trends along rows followed by trends along columns. The next level of wavelet transform is applied to the low frequency sub band image LL only. The Gaussian noise will nearly be averaged out in low frequency wavelet coefficients [8].

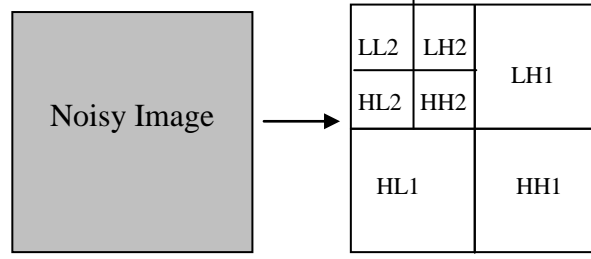


Figure -1. Two level Discrete Wavelet Transformation

All digital images contain some degree of noise [9]. The proposed de-noising algorithm attempts to remove the Gaussian noise from an image. Ideally, the resulting de-noised image will not contain more noise. De-noising of natural images corrupted by Gaussian noise using wavelet techniques is very effective because of its ability to capture the energy of a signal in few energy transform values. The methodology of the discrete wavelet transform based image de-noising has the following three steps:

- 1) Transform the noisy image into frequency domain by discrete wavelet transform.
- 2) Apply Wiener filter on each sub-band, by using local window  $n \times n$
- 3) Perform inverse discrete wavelet transform to obtain the de-noised image.

### 3. Estimate Parameters for Wiener Filter

The Wiener filter is used to removing Gaussian noise from a corrupted image based on statistics estimated from a local neighborhood of each pixel [1]. This filter depending on the noise power (i.e. noise variance in a corrupted image). Where the variance is large, the filter performs little smoothing [2,3]. Where the variance is small, the filter performs more smoothing; this filter often produces better results than other filtering used for image enhancement [5]. In this paper we using this filter with local window  $n \times n$  applied on the DWT to remove Gaussian noise from each subband. The Wiener filter illustrated in the following equations [5]:

$$\mu = \frac{1}{n^2} \sum_{i=1}^n \sum_{j=1}^n f(i, j) \quad (1)$$

$$\sigma^2 = \frac{1}{n^2} \sum_{i=1}^n \sum_{j=1}^n f^2(i, j) - \mu^2 \quad (2)$$

Note \ Mask size  $n=3,5,7$

In the above two equations used to estimates local mean and local standard deviation, for local window  $n \times n$  at noisy sub-band. Finally apply Wiener filter for each coefficient inside local window  $n \times n$  to obtain de-noised coefficients, then the local window moved one step from left to right to obtaining new de-noised coefficients  $D(i,j)$  shown in the following equation [10]:

$$D(i, j) = \mu + \frac{\sigma^2 - \text{Noise}}{\sigma^2} (f(i, j) - \mu) \quad (3)$$

Note \ i , j=3,5,7

The following **List-1** illustrated Wiener filter applied to one of the discrete wavelet transform sub-bands, assume the sub-band  $f(i,j)$ :

**List-1**

```

For i=1 to column size of a subband)
  For j=1 to row size of a subband)
     $\mu$  = Compute mean for local window(i,j);
     $\sigma^2$  = compute Standard deviation for local window(i,j);
    For  $L_1=i$  to  $i+n$ 
      For  $L_2=j$  to  $j+n$ 
         $D(L_1, L_2) = \mu + [(\sigma^2 - \text{Noise}) / \sigma^2] * [f(L_1, L_2) - \mu]$ 
      End; // for4
    End; // for3
  j=j+1 //
End; // for2
  i=i+1
End; // for1

```

In equation (3) the "Noise" is represented power of noise in corrupted image, and this value estimated by using the following equations:

$$\text{Average}(i,j) = \frac{1}{n^2} \sum_{L1=0}^{n-1} \sum_{L2=0}^{n-1} f(i + L1, j + L2) \quad (4)$$

$$\text{Variance}(i,j) = \frac{1}{n^2} \sum_{L1=0}^{n-1} \sum_{L2=0}^{n-1} f^2(i + L1, j + L2) - \text{Average}^2(i,j) \quad (5)$$

Note \ i =1,2,3,..... N  
j =1,2,3,..... M

$$\text{Noise} = \frac{1}{NM} \sum_{i=1}^N \sum_{j=1}^M \text{Variance}(i, j) \quad (6)$$

The new matrix  $\text{Average}(i,j)$  store the average value for each local window  $n \times n$  from the noisy image. The idea for this matrix look like image enhancement filter [12], uses local window  $n \times n$  to sum all coefficients, then divided by  $n^2$ . Also in same way compute the matrix  $\text{Variance}(i,j)$ , but in this matrix we take square for each coefficient then subtracted with square of the  $\text{Average}(i,j)$ , finally from this matrix compute the average value which is represents noise power (Noise). The **List-2** and **List-3** illustrated the computation for matrices;  $\text{Average}(i,j)$  and  $\text{Variance}(i,j)$  respectively for each sub-band, just the noise power for LL2 is divided by "n" which is represents root square of the  $n^2$ .

**List-1**

```

For i=1 to column size of a subband)
  For j=1 to row size of a subband)
    Sum=0;
    For  $L_1=i$  to  $i+n$ 

```

**List-2**

```

For i=1 to column size of a subband)
  For j=1 to row size of a subband)
    Sum=0;
    For  $L_1=i$  to  $i+n$ 

```

```

For  $L_2=j$  to  $j+n$ 
  Sum=Sum +  $f(i+L_1, j+L_2)$ ;
End; // for4
End; // for3
Average(  $i, j$  )= $\text{Sum}/n^2$ ;
 $j=j+1$  //
End; // for2
 $i=i+1$ 
End; // for1

```

```

For  $L_2=j$  to  $j+n$ 
  Sum=Sum +  $f^2(i+L_1, j+L_2)$ ;
End; // for4
End; // for3
Variance(  $i, j$  )= $(\text{Sum} / n^2) - \text{Average}^2(i, j)$ ;
 $j=j+1$  //
End; // for2
 $i=i+1$ 
End; // for1

```

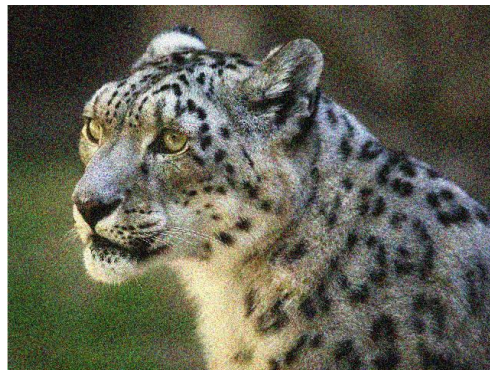
#### 4. Computer Results

In this section we applied our algorithm on the two color images (Animal and X-ray), with two noise variances. The two color images consist from three layers; Red, Green and Blue. Each layer process by implementing our algorithm using MATLAB language, the noisy images are transformed by two levels DWT to produce seven sub-bands (See Figure-1), and then apply the Wiener filter on each sub-band with two local window sizes.

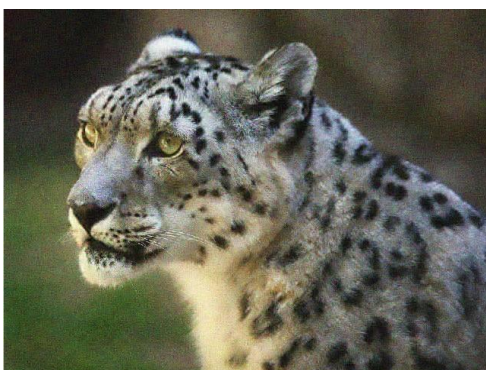
The two original images corrupted with Gaussian noise variance=0.02, and the noisy images are enhanced with our algorithm by use two local window size =  $3 \times 3$  and  $5 \times 5$ . Also the two images are corrupted with Gaussian noise variance=0.05, and our algorithm attempted to remove the noise as shown in Figure- 3.



(a) Original **Animal** image



(b) Noisy **Animal** image with Variance = 0.02

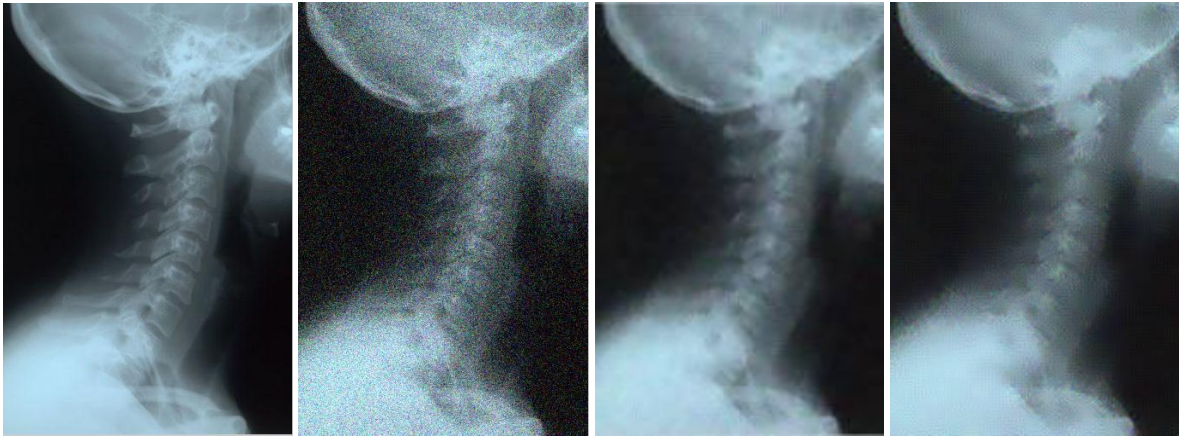


(c) Filter **Animal** image by our algorithm  
With Local window  $3 \times 3$



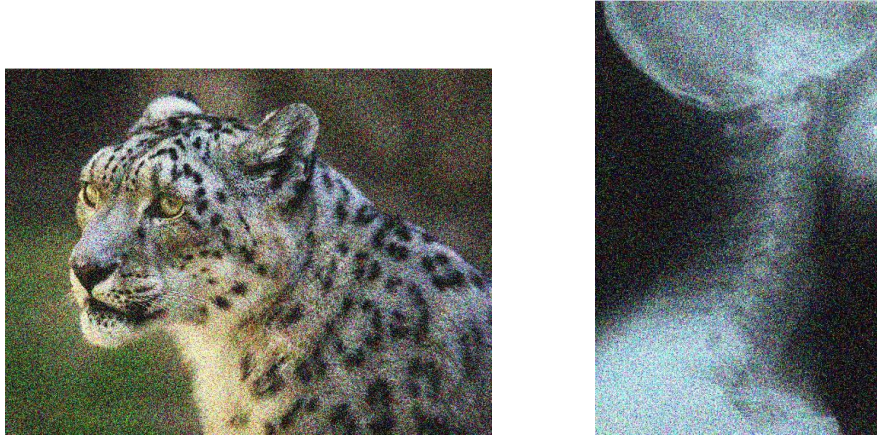
(d) Filtered **Animal** image by our algorithm  
With Local window  $5 \times 5$





(e) Original **X-ray** image      (f) Noisy **X-ray** image with Variance = 0.02      (g) Filter **X-ray** image with Local window  $3 \times 3$       (h) Filter **X-ray** image with Local window  $5 \times 5$

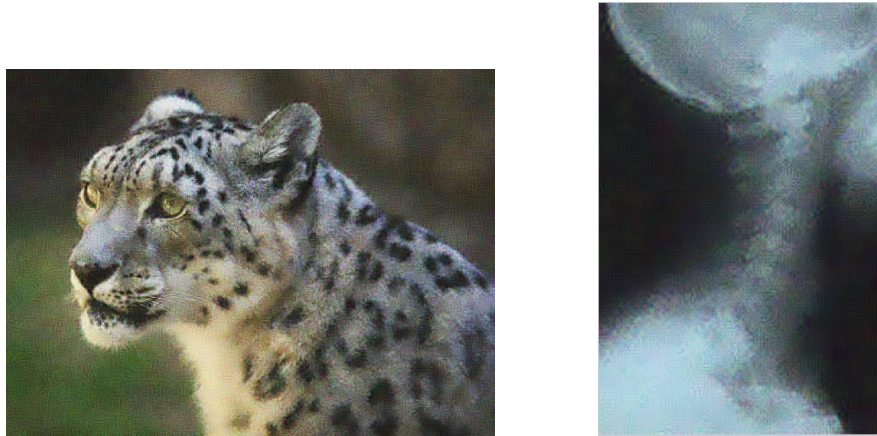
**Figure-2.** Our algorithm removed Gaussian noise from the two color images.



(a) Noisy image with Variance = 0.05



(b) Filter image by our algorithm with Local window  $3 \times 3$



(c) Filter image by our algorithm with Local window  $5 \times 5$

**Figure-3.** Our algorithm remove Gaussian noise from the two color images.

Table -1 and Table -2 shows the image quality for our algorithm by using Peak Signal to Noise Ratio (PSNR) is represents a measurement for comparing the original images with the filtered images [12]. PSNR calculated using equation (7) it is measured on a logarithmic scale and is based on the mean squared error (MSE) between an original images and filtered images, relative to  $(255)^2$  (i.e. the square of the highest possible signal value in the image). Maximum value for PSNR represents better image quality.

$$\text{PSNR} = 10 \log_{10} \frac{255^2}{\text{MSE}} \quad (7)$$

**Table 1.** Our algorithm results for **Animal Image**

Noise Variance	Corrupted Image	Filtered image Local window $3 \times 3$	Filtered image Local window $5 \times 5$
0.02	PSNR =17.4 dB	PSNR =23.2 dB	PSNR =25.1 dB
0.05	PSNR =13.9 dB	PSNR =20 dB	PSNR =22.2 dB

**Table 2.** Our algorithm results for **X-ray Image**

Noise Variance	Corrupted Image	Filtered image Local window $3 \times 3$	Filtered image Local window $5 \times 5$
0.02	PSNR =17.6 dB	PSNR =28.46dB	PSNR =29.6 dB
0.05	PSNR =14.1 dB	PSNR =24 dB	PSNR =25.1 dB

Our algorithms compared with two important filters; Normal Shrink filter and Wiener filter, they are used for noise removal, these two filters applied on the same images, for showing the performance for our algorithm. The Normal Shrink filter based on the two levels DWT and soft-threshold for each sub-band except the LL2 sub-band, this technique is widely used for removing Gaussian noise [7]. Also the Wiener filter is applied on the corrupted image with local window  $5 \times 5$ . The corrupted image in Figure -2(b,f) and Figure-3(a) filtered by Normal Shrink filter as shown in Figure-4(a) and Figure-4(b) respectively, also the result of the Wiener filter shown in Figure-4(c) and Figure-4(d) respectively.





(a) Remove the noise variance=0.02  
by using Normal Shrink filter



(b) Remove the noise variance=0.05  
by using Normal Shrink filter



(c) Remove the noise variance=0.02  
by using Wiener filter





(d) Remove the noise variance=0.05 by using Wiener filter

**Figure -4** Filtered image with conventional method

**Table 3.** Comparison with our algorithm using **Animal** image

Method	Corrupted image with Variance=0.02	Filtered image	Corrupted image with Variance=0.05	Filtered image
Our Algorithm	PSNR =17.4 dB	PSNR =26.15 dB	PSNR=13.9 dB	PSNR =23.85 dB
Normal Shrink	PSNR =17.4 dB	PSNR =25.2 dB	PSNR=13.9 dB	PSNR =23 dB
Wiener Filter	PSNR =17.4 dB	PSNR =26 dB	PSNR=13.9 dB	PSNR =23.7 dB

**Table 4.** Comparison with our algorithm using **X-ray** image

Method	Corrupted image noise Variance=0.02	Filtered image	Corrupted image noise Variance=0.05	Filtered image
Our Algorithm	PSNR =17.6 dB	PSNR =29.6 dB	PSNR=14.1 dB	PSNR =25.1 dB
Normal Shrink	PSNR =17.6 dB	PSNR =28.7 dB	PSNR=14.1 dB	PSNR =24 dB
Wiener Filter	PSNR =17.6 dB	PSNR =27.2 dB	PSNR=14.1 dB	PSNR =22.9 dB

## 5. Conclusion

In this research we present a method for image de-noising by using; two levels DWT and Wiener filter, the computer results shows our algorithm gives best image quality more than the conventional methods used in this research (See Table-1,2). Our algorithm remove all the noise approximately, especially if the noise variance more than 0.02 (See Figure -2 and Figure-3). Also in this research we are used two local window size, for test our algorithm, the results shows  $5 \times 5$  window gives better results more than other local window  $3 \times 3$  is used, and the results shows our algorithm not making blurred image compared with Wiener filter. The conventional methods ( i.e. Normal Shrink and the Wiener filters) used for image de-noising they are not remove all the noise, which means the enhanced image still infected by noise, but These methods be more efficient if the noise variance is smaller than 0.02 [1,12] (See Figure -4(a,c)). In another side the conventional methods faster than our algorithm, because our algorithm need more computation to estimate noise power, filtering by local window, and two level decomposition by DWT.

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